
OPENCURVE

A BIT OF MATHS AND PHYSICS. FOR EVERYONE.

MINUS MINUS AND NEGATIVE TIMES NEGATIVE

by @kjrnia
2019-02-25

Minus minus is plus. And negative times negative is positive. Two negatives make a positive. You may have heard or uttered these expressions many times. Even though you will know this already, here you will find an algebraic proof, just for your reference. Requirements: simple algebra from the second year in secondary, high or grammar school.



Minus minus is plus

We all should have learnt in high school that subtracting a negative number is the same as adding the positive version of that number. For example:

$$1 - (-2) = 1 + 2 = 3.$$

In human English language, it should sound something like: ‘One minus minus two equals one plus two equals three’.

Now, using just variables instead of numbers, we can write this as

$$a - (-b) = a + b,$$

where a and b are any real number.

Okay, so let's prove that, shall we? Or shall we...? Well, not yet. Let's first pretend the *opposite* is true. Suppose,

$$a - (-b) = a - b.$$

Subtracting a from both sides, we get

$$-(-b) = -b.$$

Just to add some clarity, I'm going to slap some brackets around $-b$ on the right hand side:

$$-(-b) = (-b).$$

You can see now, we have a contradiction. I mean, just in case it's not quite clear yet, let's suppose $(-b) = c$, so, replacing $(-b)$ with c , we get

$$-c = c,$$

which is, clearly, in this universe, utterly ridiculous. I mean, $-1 = 1$? I think not. So, our original statement must be true. Chin-chin, pour some glasses.

Negative times negative is positive

We also learnt in high school that multiplying a negative number with another negative number equals some positive number. So, we will prove that

$$(-a) \times (-b) = a \times b,$$

where a and b are any real number. (I put the negative numbers $-a$ and $-b$ between brackets for better visibility, not because they represent any extra information or some sort of an afterthought in the literalistic sense, which this sentence totally does.)

Mathematicians are true masters of multiplying almost anything at almost any time and even manage to get paid for it. They can truly be a productive lot sometimes. So, of course, for their employer's money's worth, they will almost never bother to properly write down the '×'-sign. Hence, we write the above equation as if we were actually earning an honest living:

$$(-a)(-b) = ab.$$

The following may seem obvious but bear with us. It's just the first step. Have a look at this tautology:

$$(-a)(-b) = (-a)(-b).$$

Okay, so far, so obvious. Now, let us add a term without disturbing the essence of the expression:

$$(-a)(-b) = (-a)(-b) + 0.$$

Still a true thing, right? Now, what is also true: anything multiplied by zero equals zero. So, let's rewrite the expression as follows:

$$(-a)(-b) = (-a)(-b) + a \times 0,$$

or, to be a little more pedantic about notation, we could write it more compactly as

$$(-a)(-b) = (-a)(-b) + a(0).$$

Now, let us replace the number 0 by variables—just the variables we are using here, to be exact. So, let's say...

$$(-a)(-b) = (-a)(-b) + a \underbrace{(b - b)}_{=0}.$$

Let's now get rid of the brackets in the last term. We do this by multiplying out the last term after the '+'-sign.

$$(-a)(-b) = (-a)(-b) + ab + a(-b).$$

Let's swap the order of the last two terms. The next step becomes easier to see. So, swapping term two with term three, yields

$$(-a)(-b) = (-a)(-b) + a(-b) + ab.$$

Now we can comfortably look at the first two terms:

$$(-a)(-b) = \underbrace{(-a)(-b) + a(-b)}_{\text{look at this comfortably}} + ab.$$

Remember how to factorise? After factorising, something like $pq + pr$ becomes $p(q + r)$, for example. Guess what, we can do the same for the two terms above, but with $(-b)$ instead:

$$(-a)(-b) = (-b)((-a) + a) + ab.$$

Now, look at the term between the large brackets. This is gorgeous, because, indeed, it amounts to 0. And anything multiplied by 0 is 0. So, what is left, is

$$(-a)(-b) = +ab.$$

Here's how, old friend. Cheerio.